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Book of Abstracts

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University of Latvia

BEHAVIOR OF SYSTEM OF PIECEWISE LINEAR DIFFERENCE EQUATIONS WITH MANY PERIODIC SOLUTIONS

INESE BULA

Department of Mathematics, University of Latvia
Jelgavas street 3, Riga LV-1004, Latvia
Institute of Mathematics and Computer Science, University of Latvia
Raina blvd. 29, Riga LV-1459, Latvia
E-mail: `inese.bula@lu.lv`

We consider the global behavior of the system of first order piecewise linear difference equations:

$$\begin{cases} x_{n+1} = |x_n| - y_n - b, \\ y_{n+1} = x_n - |y_n| - d, \end{cases} \quad n = 0, 1, 2, ..., (x_0, y_0) \in \mathbf{R}^2, \tag{1}$$

where parameters b and d are any positive real numbers. The special case with $b = d = 1$ is considered in dissertation of W. Tikjha [1].

In general case there exist an unstable equilibrium $(d; -b)$. It has been shown that there are no solutions with period 2, 3 and 4, but depending on the values of parameters b and d there are solutions with periods 5, 6, 7, 11, 12, 13, 16, 17, 18, 19, 20, 24, 27, 30, 36. Other types of periodic points have not been proven, they have not been seen in numerical experiments, but they probably exist. We have a hypothesis that all solutions are periodic or eventually periodic solutions of system (1).

The obtained results have been accepted for publication in the article [2].

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HYERS ULAM STABILITY OF NONLINEAR VOLTERRA INTEGRODIFFERENTIAL EQUATIONS ON TIME SCALES

SHRADDHA CHRISTIAN

Institute of Applied Mathematics, Riga Technical University

Zunda embankment 10, Riga LV-1048, Latvia

E-mail: `Shraddha-Ramanbhai.Christian@rtu.lv`

Consider Volterra integrodifferential equation on an arbitrary unbounded time scale $\mathbb{T}$

$$x^{\Delta}(t) = f(t) + \int_a^t K(t, s, x(s), x^{\Delta}(s))\Delta s, \quad x(a) = x_0.$$

We define a new Volterra integral equation,

$$z(t) = F(t) + \int_a^t k_1(t, s, z(s))\,\Delta s, \quad a, t \in I_{\mathbb{T}} = [a, +\infty) \cap \mathbb{T}, \tag{1}$$

where $z\colon I_{\mathbb{T}} \to \mathbb{R}^{2n}$ is the unknown function, and $k_1 : I_{\mathbb{T}} \times I_{\mathbb{T}} \times \mathbb{R}^{2n} \to \mathbb{R}^{2n}$ be rd-continuous in its first and second variable. Equation (1) is known as a Volterra integral equation on time scales. Let $F\colon I_{\mathbb{T}} \to \mathbb{R}^{2n}$, $L : I_{\mathbb{T}} \to \mathbb{R}$ be rd-continuous, $\gamma > 1$ and $\beta = L(s)\gamma$. If

$$|k_1(t, s, p) - k_1(t, s, q)| \le L(s)|p - q|, (p, q) \in \mathbb{R}^{2n}, \quad s < t \tag{2}$$

$$m = \sup_{t \in I_{\mathbb{T}}} \frac{1}{e_{\beta}(t, a)} \left| F(t) + \int_a^t k_1(t, s, 0)\Delta s \right| < \infty, \tag{3}$$

then the integral equation (1) has a unique solution $z \in C^1_{\beta}(I_{\mathbb{T}}; \mathbb{R}^{2n})$. Let $C^1_{\beta}(I_{\mathbb{T}}; \mathbb{R}^{2n})$ be the linear space of rd-continuous functions such that,

$$\sup_{t \in I_{\mathbb{T}}} \frac{\max(|x(t), |x^{\Delta}(t)|)}{e_{\beta}(t, a)} < \infty.$$

We also prove that equation (1) is Hyers-Ulam stable.

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SIMULATIONS OF BIOMASS COMBUSTION DEPENDING ON POROSITY

MĀRIS GUNĀRS DZENIS

Department of Mathematics, University of Latvia

Jelgavas street 3, Riga LV-1004, Latvia

E-mail: maris_gunars.dzenis@lu.lv

Our objective is to create mathematical model of biomass thermal decomposition. To simulate thermal decomposition of straw, wood and peat mixtures with different microwave pretreatments we define every biomass as three organic compounds. Reactivity changes off biomass microwave pretreatment we estimate using porosity. Biomass thermal decomposes as a volatile part and carbon part [2]. Reactions and coal combustion

$$C + \theta O_2 \to 2(1-\theta)CO + (2\theta - 1)CO_2$$

are modeled according to Arrhenius kinetics [1]. Gases we model using the Darcy law and mass balance equation. Numerical solutions were found using finite deference scheme and finite volume method in program MatLab.

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DIMENSIONALITY REDUCTION FOR SYMMETRIC AND SYMPLECTICITY-PRESERVING NEURAL NETWORKS

DĀVIS KALVĀNS[1], JĀNIS BAJĀRS[2]

[1] *Department of Mathematics, University of Latvia*
Jelgavas street 3, Riga LV-1004, Latvia
E-mail: davis.kalvans@lu.lv
[2] *Department of Mathematics, University of Latvia*
Jelgavas street 3, Riga LV-1004, Latvia
E-mail: janis.bajars@lu.lv

Data-driven approaches employing structure-preserving algorithms, such as symplecticity-preserving neural networks known as SympNets [1], have gained traction for learning Hamiltonian systems' dynamics. Despite their promising results, the challenge of high dimensionality persists. In this study, we investigate dimensionality reduction techniques to model Hamiltonian systems effectively in lower-dimensional subspaces, thereby reducing training times while preserving prediction accuracy.

We focus on learning nonlinear localized discrete breathers in a one-dimensional crystal lattice model, employing four dimensionality reduction techniques: Proper Orthogonal Decomposition (POD) [2], COT [3], Complex Single Value Decomposition (cSVD) [3], and Nonlinear Programming Decomposition (NLP) [3].

Moreover, we explore modifications to imbue neural networks with symmetry, a characteristic inherent to Hamiltonian system flows ($\phi_h = \phi_{-h}^{-1}$). Our results demonstrate that incorporating symmetry enhances model predictions and their stability even further.

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BEHAVIOR OF SYSTEMS OF DIFFERENCE EQUATIONS WITH MANY PERIODIC SOLUTIONS

JETE LŪCIJA KRISTONE

Department of Mathematics, University of Latvia

Jelgavas street 3, Riga LV-1004, Latvia

E-mail: `jetelucija@gmail.com`

Ladas posted an open problem about the generalized Lozi map and Gingerbreadman maps to a system that were mentioned in [1]:

$$\begin{cases} x_{n+1} = |x_n| + ay_n + b, \\ y_{n+1} = x_n + c|y_n| + d, \end{cases} \quad n = 0, 1, 2, ..., (x_0, y_0) \in \mathbf{R}^2,$$

where parameters a, b, c and d are in $\{-1, 0, 1\}$.

In our presentation we analyze behavior of systems

$$\begin{cases} x_{n+1} = |x_n| - y_n + b, \\ y_{n+1} = x_n - |y_n| + d, \end{cases} \quad n = 0, 1, 2, ..., (x_0, y_0) \in \mathbf{R}^2, \tag{1}$$

and

$$\begin{cases} x_{n+1} = |x_n| - y_n - b, \\ y_{n+1} = x_n - |y_n| + d, \end{cases} \quad n = 0, 1, 2, ..., (x_0, y_0) \in \mathbf{R}^2. \tag{2}$$

An unstable equilibrium point is possible for both systems. Then in the first case (1) there is a cycle with a period of 3, but in the second case (2) depending on the values of b and d there are cycles with different periods.

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MATHEMATICAL REPRESENTATION OF THE COMBUSTION PROCESS AND APPLICATION

LAURA LEJA[1], ULDIS STRAUTIŅŠ[2]

[1] *University of Latvia*

Raina blvd. 29, Riga LV-1459, Latvia

[2] *Institute of Mathematics and Computer Science, University of Latvia*

Raina blvd. 29, Riga LV-1459, Latvia

E-mail: `ll13172@edu.lu.lv, uldis.strautins@lu.lv`

Focusing on isotropic heterogeneous materials including wood, wood briquettes, and pellets are emerging as a sustainable substitute for fossil fuels in various uses, including residential heating. Considering the heat and mass transfer in the pellets and the need for low computational power in controlling devices, simplified mathematical models are necessary.

This study concentrates on creating a network model to simulate the thermal conversion of heterogeneous biomass pellets effectively in gasification and combustion scenarios as well as further research into the flame and its combustion control. The proposed network model facilitates the simulation of heat and mass transfer in porous media. It conceptualizes the diverse granular components (like wood and straw) as nodes in a network, interconnected by edges representing common surfaces, facilitating heat and mass exchange. To simulate the thermal decomposition of biomass, the model employs reactions with Arrhenius kinetics. It assumes the gas is an incompressible ideal fluid permeating through a porous structure.

A pressure correction approach is integrated for accurate mass transfer modeling. The model also incorporates conductive and convective heat transfer equations, complete with source terms, for precise temperature prediction. This network model is not only robust in itself but also adaptable as a component in more complex biomass conversion models. We look at an open fire - a flame, methods for burning a flame have already been introduced and developed [1] and our goal is to use Phiflow [https://tum-pbs.github.io/PhiFlow/] to control the flame in the desired shapes.

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