

# Families of Perfect Polyiamonds as Formal Languages

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**Definition 1** *A polyiamond is a simple polygon made of equilateral triangles. A perfect  $n$ -polyiamond[1] is a polyiamond whose sides are of lengths  $n, n-1, \dots, 1$  in this order. A perfect acute polyiamond has all interior angles  $60^\circ$  or  $300^\circ$ , a perfect obtuse polyiamond has all angles  $120^\circ$  or  $240^\circ$ . Perfect polyiamonds can be represented as strings over the alphabet  $\Sigma = \{a, b, c, d, e, f\}$  listing their side directions in decreasing length order.*

**Proposition 2** *There is no infinite set of perfect polyiamonds that is a regular language over  $\Sigma$ . (Hint: Apply Pumping lemma.)*

**Proposition 3** *There is an infinite set of perfect polyiamonds generated by (1) a context-free grammar; (2) a TAG grammar [2]. (Examples in [3].)*

Perfect polyiamonds $S \rightarrow \text{acecPdb},$ $P \rightarrow \text{eceaPabcb} \mid \text{eceaefabcb}.$	Acute perfect polyiamonds $S \rightarrow \text{acaeaePeacacac},$ $P \rightarrow \text{aececaceceP}(\text{caea})^5 \mid \text{cecececa}.$
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**Proposition 4** *There is a parallel context-free language (PCL)[4] generating an infinite two-dimensional family of perfect polyiamonds.*

## References

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