APE-Shapes – Polyominoes, Polyiamonds and Polyhexes

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The term APE (proposed by second author) is formed from the first letters of the words area, perimeter, and edge and mathematically means the equality of three quantities $A = P = E$. The term 'equable shapes' is used in the literature to denote shapes for which $A = P$. A two-dimensional equable shape is one whose area is numerically equal to its perimeter [1], [2].

The Baltic Way competition in Flensburg, November, 2023, offers a problem involving the path of a robot in the plane related to a polyomino with $P = E$. In [3] the problem is formulated as follows: 'A robot moves in the plane in a straight line, but every one meter it turns 90° to the right or to the left. At some point it reaches its starting point without having visited any other point more than once, and stops immediately. What are the possible path lengths of the robot?' This problem is a simplified version of the problem submitted by A. Cibulis, which asked to find a polyomino with minimum area A for which $A = P = E$.

The talk will deal with the question of what values the number of edges can take for APE-polygons on square, triangular and hexagonal grids, i.e., polyominoes, polyiamonds and polyhexes, respectively. In the case of polyiamonds, the following result holds: The smallest APE-polyiamond is obviously the unit hexagon, and from $n = 10$ onwards, there exist APE-polyiamonds for which $A = P = E = n$. In the case of polyominoes and polyhexes it is more challenging to obtain valid values of $A = P = E$.

References

[1] Bradley, Christopher J.: *Challenges in Geometry: For Mathematical Olympians Past and Present.* Oxford UP., (2005), 205 p.

[2] https://en.wikipedia.org/wiki/Equable_shape

[3] https://balticway2023.de/wp-content/uploads/2024/01/BW23_Solutions.pdf