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Fuzzy Logic Based Mathematical Structures and Their Applications

Book of Abstracts

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Contents

Applications of Geometric Interpretation in Proving Quantum Strategy Optimality in XOR Games

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Nonlocal games model cooperative multiplayer games, wherein players are allowed to strategize collectively before the game, but then during the game they must act based on their limited, local information of the game state. It turns out that players can improve their chances of winning by utilizing local measurements of a shared, entangled quantum state as part of their strategy.

We begin with a brief description of a specific class of nonlocal games called XOR games, what constitutes a quantum strategy in these games, and how the analysis of these strategies relates to upper bounds for sums of norms in Hilbert spaces.

Afterward, we discuss how geometric considerations can be used to guide the process of proving sharp upper bounds and provide examples. We apply the results from the examples to prove the optimality of some quantum strategies.

References:

[1] Faleiro R.: *Quantum strategies for simple 2-player XOR games.* Quantum Information Processing **19** (2020) 229.

Properties of Quantum Logic Maps on a Set of Symmetric Binary Matrices

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A quantum logic is one of possible mathematical models for non-compatible random events. In the work we solve a problem proposed in the conference FSTA 2006 [1]. Namely, it is proved that s-maps are symmetric fuzzy relations on a set of all symmetric binary matrices. We examine properties of s-maps and of other quantum logic maps, such as j-maps and d-maps [2] based on the same conditions. We mention ways to generalize s-maps for non-binary matrices by incorporating t-norms and other binary operations for a definition of a matrices product.

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Fuzzy Relations

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This work is devoted to the definition and study of fuzzy equivalence and inequality relations ([3], [1]) in the context of fuzzy logic [2]. We begin by introducing the fundamental concepts of fuzzy logic and then delve into the construction of these principles using additive generators. Specifically, we explain the process of generating Archimedean t-norms and t-conorms through their additive generators.

Subsequently, we formally define fuzzy equivalence and inequality relations and illustrate their construction by employing the additive generators of t-norms and t-conorms, respectively. A significant aspect of our study involves investigating how these fuzzy relations are influenced by input parameters, shedding light on their dynamic behavior.

Our research contributes to a deeper understanding of fuzzy set theory and its practical applications, offering insights into the manipulation and utilization of fuzzy equivalence and inequality relations across various domains.

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Adjoint Functors in Fuzzy Category Theory

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Fuzzy category theory describes category-like structures in which potential objects and potential morphisms are respectively objects and morphisms only to a certain degree [1]. By easing the requirements on objects and morphisms, we can create models for situations that cannot be directly described using the tools of classical category theory, whilst also offering a local description of objects and morphisms in an otherwise context-free setting. This also allows us to look at categories with crisp objects and morphisms from the point of fuzzy set theory without necessarily framing, explicitly or implicitly, the objects or morphisms of a category in a fuzzy way.

Building on the work done on functors in fuzzy categories [2], we further develop the theory until the notion of adjoint functors using the unit and co-unit. We begin by defining natural transformations in fuzzy categories, which then leads us to the definition of adjoint functors. We also provide a sufficient condition for a fuzzy functor so that it admits a left adjoint with a certain degree.

By finding a pair of fuzzy adjoint functors we also obtain a class of crisp adjoint functors in well-known categories when we restrict ourselves to a threshold category. We will provide examples that naturally generalize crisp adjoint functors resulting in fuzzy adjoint functors.

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On Some Properties of Aggregation Operators Used for Data Assessment

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Aggregation operators provide various options for their construction and application in different areas. The main concepts of aggregation operators have been widely analyzed in, e.g. [1]. We consider two cases when the results of aggregation should meet certain predefined criteria, analyze the properties of particular aggregation operators and introduce weighted aggregation proposed in [2]. In the first case, we consider commutative non-decreasing aggregation operator for ensuring that the aggregated result is greater than or equal to the value of smallest element in the set of given values. Such approach has been considered in our previous research [3] outlining the application of these aggregation operators in risk assessment. In the second case, we consider non-increasing aggregation operators. For the purposes of this analysis the aggregated result should be less than or equal to the value of greatest element in the set of given values. This method is further applied in the comparison of information provided in two different datasets.

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Aggregation of Fuzzy Equivalence Relations in Hierarchical Clustering Algorithm

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In the clustering process, one of the important measures for grouping data into different clusters is the similarity and the dissimilarity of two objects. If the dissimilarity can be obtained by using Euclidean, Manhattan, Hamming, or other distances, the similarity of two objects is inversely proportional to the dissimilarity and can be expressed via the distance between two objects in different ways:

$$s(x, y) = e^{-d(x,y)}, \quad s(x, y) = \frac{1}{1 + d(x, y)}.$$

Such similarity measures are special cases of fuzzy equivalence relations, namely fuzzy equivalence relations, where transitivity is fulfilled for the product and Hamacher t-norm.

Previously, in [1], it was shown how fuzzy equivalence relations and the aggregation of corresponding equivalence relations were involved in the clustering process. This work considers hierarchical clustering, and fuzzy equivalence relations for different t-norms are considered as the similarity measures for objects and clusters. With such a similarity measure, it is possible to obtain measures of clustering performance, such as a potential function that shows whether the clusters are well separated or not. The work provides a comparative analysis of clustering results using different t-norms for fuzzy equivalence relations.

References:

[1] Grigorenko O., Mihailovs V: *Aggregated fuzzy equivalence relations in clustering process.* Communications in Computer and Information Science **1601** (2022) 448–459.

Fuzzy Morphological Operators on Additive Groups

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Mathematical morphology was initiated in 1964 in the works of G. Matheron and J. Serra [2], and was initially driven by the needs of practical geology. However, it soon found important applications in other fields, particularly in image processing, and is now widely studied and applied by many researchers. The first version of mathematical morphology in the context of fuzzy sets was presented in the paper by De Baets et al. [1]. As in the case of "classical"(i.e. crisp) mathematical morphology, the basis here is the linear structure of the Euclidean space $\mathbb{R}^n$ and a chosen subset $B \subset \mathbb{R}^n$, intuitively small and called the structuring element. They give rise (in this case fuzzy) to basic operators of mathematical morphology: erosion $\mathcal{E}$ and dilation $\mathcal{D}$. There was a lot of work done in the study of fuzzy morphological operators on "structured" Euclidean space and many applications have they found in different practical problems. However, as far as we know, there was no done any work to embed (fuzzy) mathematical morphology spaces into the categorical framework. In turn this essentially restrict the possibility to consider relations between (fuzzy) morphological spaces, their transformations, products and direct sums of (fuzzy) morphological spaces, et al.

The purpose of our talk is to present a category containing (fuzzy) morphological spaces realized in the spirit of the article [1] providing us the flexibility to deal in the framework of such objects. As the basis for this category we take additive groups $(X, +_X, 0_X)$ and $(Y, +_Y, 0_Y)$ as objects and certain L-fuzzy relations $R : X \times Y \to L$ between them as morphisms.

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[1] De Baets B., Kerre E.E., Gupta M.: *The fundamentals of fuzzy mathematical morphology Part I: Basic concepts.* International Journal of General Systems **23** (1995) 155–171.

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Fuzzy Equivalence Based Numerical Algorithm for Solving Maximin Problems

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In our previous work [1], we showed that solving multi-objective linear programming problems, the problem is reduced to

$$\max_{y \in D} min\{f_1(y), \dots f_n(y)\}, \tag{1}$$

where D is our search space, and the functions f_i correspond to the vertices x_i of the search space D. To solve problem (1), we proposed to make a linear combination of ∇f_{i_1} and ∇f_{i_2}, where weights were assigned using a fuzzy equivalence relation, and use this combination to construct the search direction. The functions f_{i_1} and f_{i_2} were chosen to be the smallest and second smallest at some point y.

In this work, we want to generalize the previously mentioned algorithm to use the method for the more general case where the optimization problem is of the form (1), without being based on any linear programming problem. Another generalization we want to make is to construct the previously mentioned search direction as a linear combination of more than two components. We illustrate the algorithm with multiple examples.

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