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Boundary Value Problems for Ordinary Differential Equations

Book of Abstracts

Institute of Mathematics and Computer Science
University of Latvia

EXISTENCE OF A POSITIVE SOLUTION FOR A BOUNDARY VALUE PROBLEM FOR SYSTEM OF SECOND-ORDER DIFFERENTIAL EQUATIONS

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We show the result on existence of at least one (strictly) positive solution for a boundary value problem for a system of two second-order differential equations

$$\begin{gathered} x'' + f(t, x, y, x', y') = 0, \quad t \in [0, 1], \\ y'' + g(t, x, y, x', y') = 0, \quad t \in [0, 1], \\ x(0) = \alpha_0[x],\ x(1) = \alpha_1[x],\ y(0) = \beta_0[y] + \gamma_0,\ y(1) = \beta_1[y] + \gamma_1, \end{gathered}$$

where $\gamma_0, \gamma_1 \geq 0$, $f : [0, 1] \times [0, +\infty)^2 \times \mathbb{R}^2 \mapsto [0, +\infty)$ and $g : [0, 1] \times [0, +\infty)^2 \times \mathbb{R}^2 \mapsto (-\infty, 0]$ are continuous, and α_i, β_i are linear functionals involving Riemann–Stieltjes integrals.

Since $f \geq 0$ and $g \leq 0$, first component of the solution (x, y) is concave and second component is convex. Gronwall-type inequality [1] is used to obtain *a priori* bounds for norm of a derivative and hybrid Krasnosel'skii-Schauder fixed point theorem [2] is used to prove the existence of a positive solution.

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ON DIFFERENTIAL EQUATIONS WITH PERIOD ANNULI

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We consider the equation

$$x'' + g(x) = 0, \tag{1}$$

where $g(x)$ is an odd degree polynomial with simple zeros [1; 2].

We add a function $f(t) = h \cdot \cos \omega t$ on the right, so the equation becomes

$$x'' + g(x) = f(t). \tag{2}$$

We study behavior of solutions in equation (2) which, without the external force, have period annuli.

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THIRD ORDER SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS WITH SADDLE-FOCUS EQUILIBRIA

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A three-dimensional multiparametric system of ordinary differential equations in a gene regulatory network is considered. The proposed model has eighteen parameters. Examples of genetic systems which have saddle-focus critical point type with chaotic behavior of solutions were constructed.

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REMARKS ON MATHEMATICAL MODELING OF GENE AND NEURONAL NETWORKS

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We consider the system of ordinary differential equations of the form

$$\begin{cases} \dfrac{dx_1}{dt} = \tanh(a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n - \theta_1) - b_1x_1, \\ \dfrac{dx_2}{dt} = \tanh(a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n - \theta_2) - b_2x_2, \\ \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \dfrac{dx_n}{dt} = \tanh(a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n - \theta_n) - b_nx_n, \end{cases}$$

which supposedly model artificial neuronal networks. The above system describes the evolution of these networks in time. We provide examples of systems with stable critical points, stable periodic trajectories (limit cycles), and attractors exhibiting chaotic behavior. We also make comparisons with systems, modeling genetic networks.

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ON DUFFING EQUATION

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Properties of solutions to the Duffing equation are considered. These properties relate to the notion of the index of a solution. We show that indexes of solutions change in a non-monotone way.

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THE MODIFIED GAO-MA SYSTEM

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We define the modified Gao-Ma system

$$\begin{cases} x' = z + (y-a)x, \\ y' = 1 - by - x^6, \\ z' = -x - cz, \end{cases} \tag{1}$$

where the variable x signifies the interest rate, y corresponds to the degree of investment demand, and z denotes the exponential factor of prices [1]. Additionally, the constant $a \geq 0$ signifies the household savings rate, while $b \geq 0$ represents the investment cost and $c \geq 0$ is the elasticity of demand of commercial markets [2].

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ON SOME FUČÍK PROBLEM WITH ONE BITSADZE-SAMARSKII TYPE NONLOCAL BOUNDARY CONDITION

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Consider the Fučík equation with one Bitsadze-Samarskii type nonlocal boundary condition

$$-x'' = \mu x^+ - \lambda x^-, \tag{1}$$
$$x'(0) = 0,\ x(1) = \gamma x(\xi), \tag{2}$$

with the parameters $\mu, \lambda, \gamma \in \mathbb{R}$ and $\xi \in (0,1)$.

The spectrum of the problems (1), (2) for some ξ values is investigated.

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A PRIORI ESTIMATE AND EXISTENCE OF SOLUTIONS WITH SYMMETRIC DERIVATIVES FOR A THIRD-ORDER TWO-POINT BOUNDARY VALUE PROBLEM

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We study a priori estimate and the existence of solutions with symmetric derivatives for the third-order two-point boundary value problem

$$x''' = f(t, x, x', x''),\ t \in (0, 1),$$

$$x(0) = 0,\ x(1) = 0,\ x'(t) = x'(1 - t).$$

The main tool in the proof of our result is Leray-Schauder Continuation Principle. To illustrate the applicability of the obtained results, we consider an example.

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Index of Authors

Antoņuks A., 2
Atslega S., 3

Dobkeviča M., 6

Kozlovska O., 4

Ogorelova D., 5

Sadirbajevs F., 6
Samuilik I., 7
Sergejeva S., 8
Smirnov S., 9

Urbonienė S., 8