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Boundary Value Problems for Ordinary Differential Equations

Book of Abstracts

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EXISTENCE OF A POSITIVE SOLUTION FOR A SYSTEM OF FOURTH-ORDER DIFFERENTIAL EQUATIONS WITH BOUNDARY CONDITIONS INVOLVING MULTIPLICATION

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We present a result on the existence of a positive solution for a system of two fourth-order nonlinear differential equations

$$\begin{aligned} x^{(4)}(t) + f_1\big(t, x(t), y(t)\big) = 0, \quad t \in (0,1), \\ y^{(4)}(t) + f_2\big(t, x(t), y(t)\big) = 0, \quad t \in (0,1), \end{aligned} \tag{1}$$

coupled with nonlocal nonlinear boundary conditions

$$\begin{aligned} x'(0) = x''(0) = x'''(0) = 0, &\quad x(1) + x'(1) = k_1\, x'(\alpha)\, x'''(\beta), \\ y(0) = y''(0) = y'''(0) = 0, &\quad y(1) = -k_2\, y(\gamma)\, y''(\delta). \end{aligned} \tag{2}$$

We call $(x, y) \in C^4\big([0,1]\big) \times C^4\big([0,1]\big)$ a positive solution of the problem (1),(2) if (x, y) satisfies differential equation (1), boundary conditions (2) and $x(t) > 0$, $y(t) > 0$ for every $t \in (0,1)$.

Standard method of obtaining positive solutions is to rewrite a system of boundary value problems as an equivalent system of integral equations and seek solutions as fixed points of the corresponding integral operator in suitable cone [1]. The proof of the existence of fixed points relies on the vector version of Krasnosel'skiĭ's fixed point theorem [2; 3].

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CONTROLLABILITY OF TWO DIMENSION GENE REGULATORY SYSTEM WITH SEASONALITY

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The authors in [1] summarize the progress on the molecular and genes mechanisms of seasonal regulation and seasonal changes. Authors describe how important are rhythmic cycles and that small changes in seasonality cycle can involve changes in genes.

In the articles [2], [3], [4] the authors study system which is part of the gene regulatory system.

The authors in [5] observed the seasonality function

$$r_0 = r(1 + \epsilon \sin \theta t). \tag{1}$$

In the current research we extend the two dimensional gene regulatory system

$$\begin{cases} \dfrac{dx_1}{dt} = \dfrac{1}{1 + e^{-\mu_1(w_{11}x_1 + w_{12}x_2 - \theta_1)}} - v_1 x_1, \\ \dfrac{dx_2}{dt} = \dfrac{1}{1 + e^{-\mu_2(w_{21}x_1 + w_{22}x_2 - \theta_2)}} - v_2 x_2 \end{cases} \tag{2}$$

by change the parameters w_{ij} with the simplified seasonality function

$$S(w_{ij}, \xi_{ij}, t) = F_S = w_{ij} + \sin \xi_{ij} t. \tag{3}$$

With the seasonality functions is possible to drive the system solutions from one attractor to another one. Reasons of driving solutions from one attractor to another is controllability. In the papers [6], [7] controllability use are observed. In gene regulatory system the seasonality functions in some fixed moment t can change types of solutions attractions for the gene regulatory system (2).

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ANALYSIS OF A NEW 6D HYPERCHAOTIC SYSTEM

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We consider a novel six-dimensional (6D) dynamic system derived from a modified second-type 3D Lorenz system

$$\begin{cases} \dfrac{dx_1}{dt} = a(-x_1 + x_2) + x_4, \\ \dfrac{dx_2}{dt} = -x_3 \mathrm{sgn}(x_1), \\ \dfrac{dx_3}{dt} = -1 + |x_1|, \\ \dfrac{dx_4}{dt} = -bx_1, \\ \dfrac{dx_5}{dt} = -x_5 + x_1 x_4, \\ \dfrac{dx_6}{dt} = -x_6 + x_1 x_3. \end{cases} \tag{1}$$

The newly 6D hyperchaotic system exhibits diverse dynamic behaviors. A comprehensive dynamical analysis was performed, including bifurcation diagrams, Lyapunov exponent and dimension calculations.

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CHAOTIC BEHAVIOR OF DUFFING-TYPE EQUATIONS

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We observe the modified Duffing equation

$$\begin{cases} x' = y, \\ y' = -x^3 + x + Fcos(\omega t), \end{cases} \tag{1}$$

where F is the amplitude of the periodic driving force and ω is the angular frequency of the periodic driving force. Examples with chaotic behavior of the modified Duffing equation were constructed. The chaotic behavior was confirmed by analysis using Lyapunov exponents.

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NOTES ON ATTRACTORS OF THE 4-DIMENSIONAL NEURAL SYSTEM

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We consider the four-dimensional system of ordinary differential equations that model the 4-element neural network. The 4D attractor is constructed as a product of two 2D attractors, limited cycles with equal periods.

The general system, which is used to model ANN of 4 elements, is

$$\begin{cases} \dfrac{dx_1}{dt} = \tanh\left(w_{11}x_1 + \ldots + w_{14}x_4\right) - b_1x_1, \\ \dfrac{dx_2}{dt} = \tanh\left(w_{21}x_1 + \ldots + w_{24}x_4\right) - b_2x_2, \\ \dfrac{dx_3}{dt} = \tanh\left(w_{31}x_1 + \ldots + w_{34}x_4\right) - b_3x_3, \\ \dfrac{dx_4}{dt} = \tanh\left(w_{41}x_1 + \ldots + w_{44}x_4\right) - b_4x_4. \end{cases} \tag{1}$$

The hyperbolic tangent function $\tanh(z)$ is sigmoidal, but its range value is $(-1, 1)$.

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ON A DUFFING-TYPE EQUATION

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We consider Duffing equation, which is rich of solutions behavior and allows for chaos also. Let us analyze the behavior of solutions if the cubic nonlinearity in the Duffing equation is replaced by a polynomial of a higher degree. Through numerical experiments, suitable parameter values are found to demonstrate chaotic behavior in the modified equation.

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MEETING GRN SYSTEM AND NONLINEAR OSCILLATOR

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We know that the matrix W can be variable. We have discovered previously several attractors for the system

$$\begin{cases} x_1' = \dfrac{1}{1+e^{-\mu_1(w_{11}x_1+w_{12}x_2+w_{13}x_3-\theta_1)}} - v_1x_1, \\ x_2' = \dfrac{1}{1+e^{-\mu_2(w_{21}x_1+w_{22}x_2+w_{23}x_3-\theta_2)}} - v_2x_2, \\ x_3' = \dfrac{1}{1+e^{-\mu_3(w_{31}x_1+w_{32}x_2+w_{33}x_3-\theta_3)}} - v_3x_3, \end{cases} \tag{1}$$

where the coefficients of $W(t)$ were expressed using $\sin[t]$, $\sin[t]^2$ functions.

We wish now to repeat this using lemniscate functions $sl[t]$, $\mathrm{sl}[t]^2$. The function $\sin[t]$ is a solution of the equation $x'' + x = 0$. The function $sl[t]$ is a solution of the problem $x'' + 2x^3 = 0$, $x(0) = 0$, $x'(0) = 1$, it can be expressed as $\mathrm{sl}[t] = \mathrm{sn}\left(\sqrt{2}t; \dfrac{1}{2}\right)$.

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SOME NOTES ON SPECTRA OF SOME FUČÍK TYPE PROBLEMS WITH NONLOCAL TWO-POINT BOUNDARY CONDITIONS

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Let us consider the Fučík problem

$$x'' = -\mu x^+ + \lambda x^-, \tag{1}$$

with nonlocal two-point boundary conditions of four types

$$x(0) = 0, \quad x(1) = \gamma x(\xi), \tag{2}$$
$$x'(0) = 0, \quad x(1) = \gamma x(\xi), \tag{3}$$
$$x(0) = 0, \quad x(1) = \gamma x'(\xi), \tag{4}$$
$$x'(0) = 0, \quad x(1) = \gamma x'(\xi), \tag{5}$$

where $\xi = \frac{m}{n} \in (0, 1)$ and $\gamma \in \mathbb{R}$, m and n $(0 < m < n)$ are positive coprime integer numbers.

The idea for this study and the type of boundary conditions was taken from the work [1], where the linear Sturm-Liouville equation $x'' = -\lambda x$ was analyzed with boundary conditions (2) - (5).

The aim of the study is to identify the main similarities and differences of the spectra, where $\xi = \frac{1}{2}$.

The obtained results generalize and continue of author's previous established investigations [2], [3], [4].

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EXISTENCE OF A UNIQUE SOLUTION FOR A THIRD-ORDER BOUNDARY VALUE PROBLEM WITH NONLOCAL CONDITIONS OF INTEGRAL TYPE

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We study boundary value problem consisting of the nonlinear third-order differential equation

$$x''' + f(t,x) = 0,\ t \in [a,b], \tag{1}$$

and the integral type boundary conditions

$$x(a) = 0,\ x(b) = 0,\ \int_a^b x(\xi)d\xi = 0. \tag{2}$$

The existence of a unique solution for problem (1),(2) is proved in several ways. The main tools in the proofs are the Banach fixed point theorem [3] and the Rus's fixed point theorem [1]. To compare the applicability of the obtained results, some examples are considered.

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