

SPECTRAL AND MULTIVARIATE STATISTICAL ANALYSIS OF ASTROGEODETTIC QUANTITIES DERIVED FROM GGM

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Accurate determination of the deflection of the vertical (DoV) is essential for high-precision geoid modeling and for establishing consistent height systems [1]. In this study, we evaluate the agreement between observed DoV values obtained by the digital zenith camera and those computed from several Global Geopotential Models (GGMs) obtained from the ICGEM repository [2], including EGM2008, EIGEN_6C4, EIGEN_6C3stat, GECO, XGM2019e, and SGG_UGM_2. The comparison focuses on the north–south (η) and east–west (ξ) components of the DoV across a spherical harmonic degree/order (d/o) range truncated at 1500, with an emphasis on identifying the models and spectral ranges that best represent the measured signal.

The analysis begins with the computation of residual standard deviations (STDs) between measured and GGM-derived DoV components (Figure 1). These results reveal that EGM2008 and GECO consistently exhibit the lowest residual STDs across much of the mid-frequency domain, particularly between d/o 300 and 800. These findings indicate superior agreement with the observed DoV within this spectral range. EIGEN_6C4 demonstrates comparable performance but with slightly elevated residuals at higher degrees. In contrast, models such as XGM2019e show increased variability in the residuals at finer spatial scales, potentially reflecting differences in regularization strategies, satellite-only content, or resolution limitations in the model construction.

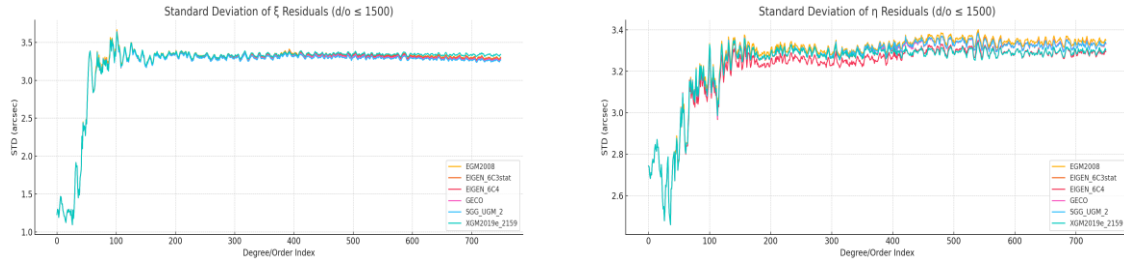


Fig. 1. Standard Deviation of residuals for $\xi\xi\eta$

To gain further insight into model-specific residual behavior, dimensionality reduction was performed using Principal Component Analysis (PCA). PCA projections of the truncated STD vectors reveal distinct clustering patterns. Notably, EIGEN_6C4 and GECO occupy a similar region of the principal component space, suggesting shared spectral characteristics in their error structures (Figure 2.a). Conversely, XGM2019e projects further from the cluster center, reflecting its relatively distinct variance structure, particularly in the ξ component [3]. Frequency-domain analysis was also employed to examine the distribution of residual energy. Discrete Fourier Transform (DFT) plots (see Figure 2.b show that the majority of residual variance is concentrated in the low-frequency range, consistent with large-scale geophysical signal content [4]. However, model-specific distinctions emerge in the mid-frequency domain. For instance, GECO and

EGM2008 exhibit smooth spectral decay, indicative of well-controlled residuals. In contrast, SGG_UGM_2 and XGM2019e display elevated spectral amplitudes in the mid- and high-frequency bands, suggesting the presence of spatially correlated errors or insufficient attenuation of high-frequency noise.

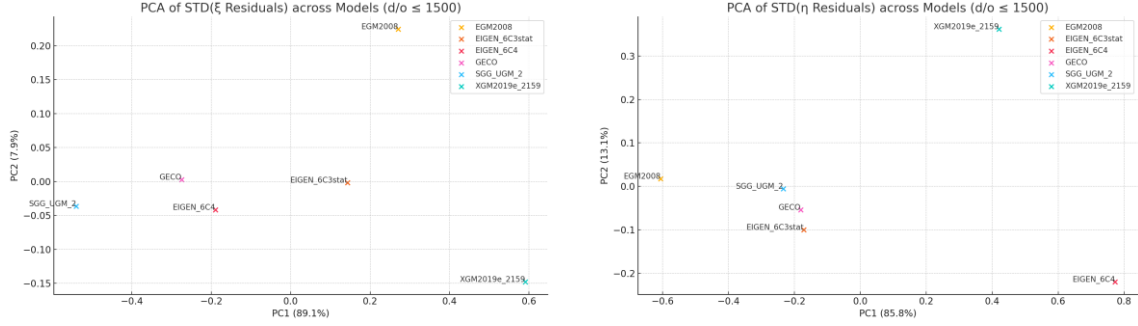


Fig. 2.a. PCA of Standard Deviation of residuals of ξ and η

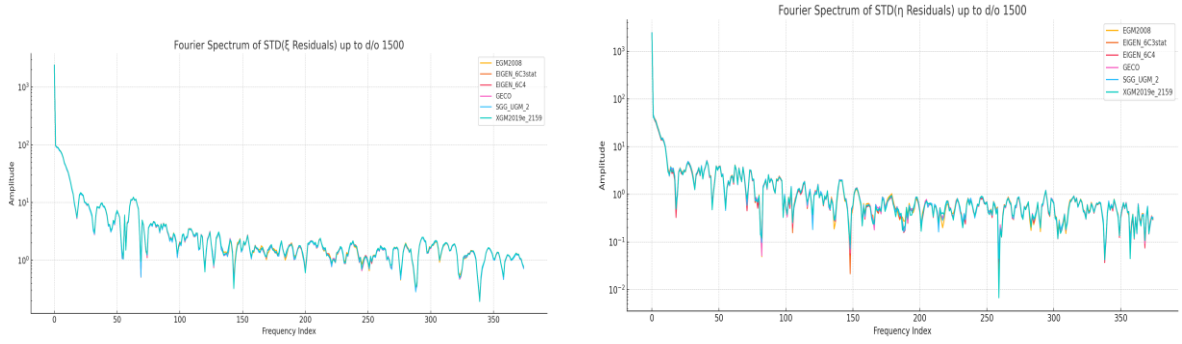


Figure 2.b. PSD of Standard Deviation of residuals of ξ and η

Complementing the DFT, power spectral density (PSD) analysis quantifies the distribution of residual energy across continuous frequency bands. The PSD plots in Figure 3 reaffirm that EGM2008 and GECCO maintain consistent energy levels with minimal high-frequency amplification, whereas the other models show less spectral smoothness. These differences may be linked to the inclusion of high-resolution terrestrial data or filtering techniques employed during model development [5].

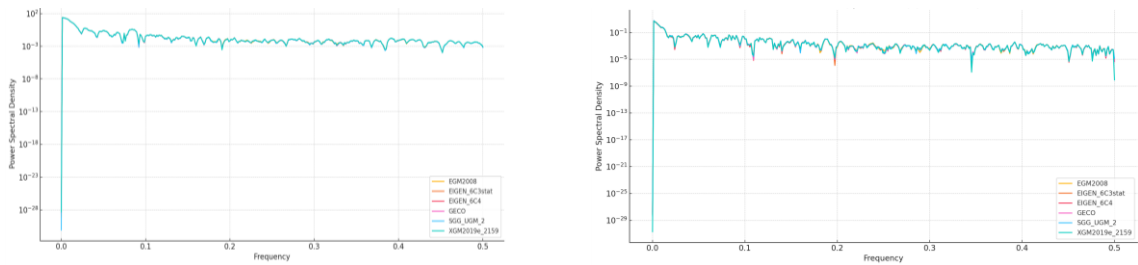


Fig. 3. PSD analysis of the residuals

Together, these multi-level analyses demonstrate that while direct residual statistics provide a baseline evaluation of GGM performance, advanced spectral and multivariate techniques offer enhanced diagnostic power. The combination of PCA, DFT, and PSD reveals model behaviors that

would not be evident from spatial-domain analysis alone. Notably, the mid-degree range (d/o 300–800) emerges as the zone of optimal model–observation agreement, suggesting its importance in model truncation strategies for regional geoid applications [6].

This work highlights the value of integrating spectral and statistical methods in the evaluation of global gravity field models, particularly when validated against high-quality terrestrial observations. Such comprehensive assessments are critical for advancing geodetic infrastructure, refining geoid estimates, and ensuring robust integration of GNSS and leveling data in national height systems.

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