# MACHINE LEARNING–ENHANCED GEOID ESTIMATION FROM TERRESTRIAL AND SATELLITE DATA

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Accurate orthometric height is a fundamental requirement in geodesy, as well as in a wide range of geophysical and engineering applications. Orthometric height is defined relative to mean sea level (MSL), which is closely approximated by the geoid, a complex, equipotential surface representing the Earth's gravity field. Traditionally, orthometric heights are determined using precise leveling, which yields height differences with high relative accuracy. However, despite its precision, this technique is labor-intensive, time-consuming, and often impractical for extensive or inaccessible regions. In contrast, modern Global Positioning System (GPS) technology enables rapid and accurate position determination, but the heights it provides are relative to a reference ellipsoid, a simplified mathematical model of the Earth's shape [1]. For practical and scientific purposes, these ellipsoidal heights must be transformed into orthometric heights to relate measurements to the physical surface of the Earth and to ensure consistency with existing geodetic and engineering frameworks.

The geoidal height, or undulation, indicates the vertical separation between the geoid and the reference ellipsoid. This measure is essential for modern height systems as it facilitates the conversion of GPS-derived ellipsoidal heights into orthometric heights. Orthometric heights are referred to as mean sea level and are commonly used in geodetic, engineering, and mapping applications. The geoid represents the Earth's gravity field and approximates mean sea level, while the ellipsoid is a smooth, mathematically defined reference surface utilized by satellite positioning systems. Determining geoidal height is crucial for integrating GPS data with traditional leveling networks. This can be accomplished using geometric methods that leverage co-located GPS and leveling data. Alternatively, gravimetric methods rely on gravity observations and physical modeling or applying Global Geopotential Models (GGMs). GGMs combine satellite and terrestrial gravity data to produce global estimates of the geoid at various resolutions. These approaches facilitate consistent height referencing at local, regional, and global scales.

GGMs represent the Earth's gravitational potential through spherical harmonic expansions, enabling the computation of geoid undulations with global coverage. These models are constructed using a combination of satellite-based gravity missions, such as the Gravity Recovery and Climate Experiment (GRACE) and the Gravity field and steady-state Ocean Circulation Explorer (GOCE), along with terrestrial and marine gravimetry, airborne gravity surveys, and satellite altimetry. GGM accuracy varies with spectral content due to factors such as spatial resolution and noise measurement. This study analyzes five recent ultra-high-degree GGMs, namely, XGM2019e, GECO, EGM2008, SGG-UGM-2, and EIGEN-6C4, obtained from the International Centre for Global Earth Models (ICGEM) [3], complete to degree and order 2190, focusing on geoid undulations from degree 5 and upwards. Machine learning techniques are used to correct discrepancies between GGM-derived undulations and control point observations, improving geoid estimation accuracy [4].

This study assesses the accuracy of GGMs by comparing gravimetric geoid undulations with geometric geoid undulations computed from GPS and leveling data at 5,379 benchmarks across the United States. The GPS/Levelling dataset used for this research is obtained from the National Geodetic Survey (NGS) website [2]. The gravimetric undulations are derived through spherical harmonic synthesis of the GGM, incorporating topographic corrections, while the geometric undulations represent observed differences between ellipsoidal and orthometric heights. Residuals between the two are analyzed as a function of spherical harmonic degree to quantify model errors. Statistical measures such as RMSE, mean bias, and standard deviation are used to evaluate each GGM's fidelity. The analysis identifies the optimal truncation degree that minimizes residuals and establishes a foundation for hybrid geoid modeling using residual interpolation or machine learning.

The results show that the lowest Root Mean Square Error (RMSE) values for the GGMs were consistently noted at a maximum degree and order (d/o) of 2140. The best RMSE was 62.75 cm from XGM2019e_2159, while the worst was 63.31 cm from EGM2008, resulting in a minimal difference of 0.56 cm across models. The other models, SGG-UGM-2 (62.93 cm), GECO (63.07 cm), and EIGEN-6C4 (63.09 cm), showed similar performance, indicating convergence at fine spectral scales. The RMSE differences between d/o 2140 and 2190 were negligible, between -0.27 mm and -1.01 mm, suggesting limited gains in predictive accuracy beyond d/o 2140. In contrast, RMSE reductions between d/o 720 and 2140 were substantial, ranging from -20.6 mm to -25.7 mm. Overall, these findings indicate that increasing GGM resolution beyond d/o 2100 offers only minor improvements relative to computational costs.

Figure 1 (Left) shows the RMSE trend of the GGM-derived geoid undulations from d/o 5 to 2190, and Figure 1 (Right) shows the log of the RMSE values from d/0 360 to 2190.

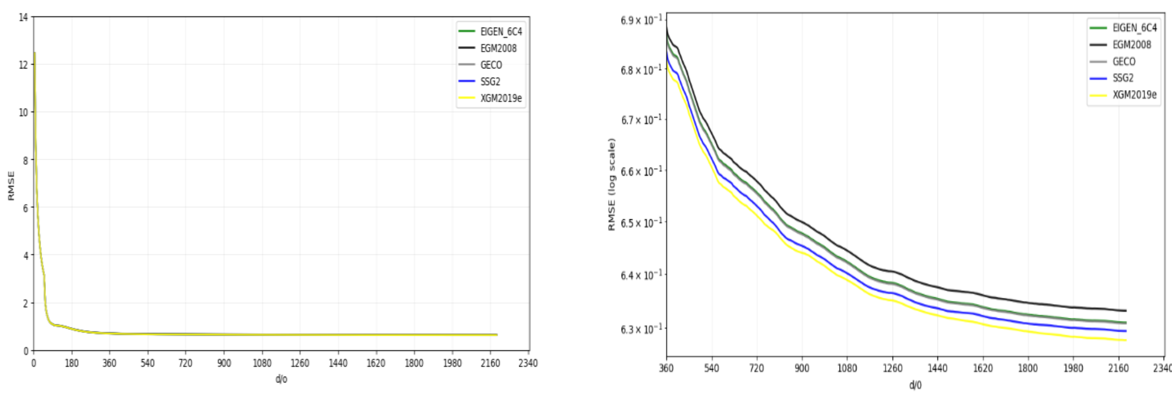


**Fig. 1.** RMSE comparison of Residuals Between GGMs and GPS/Levelling geoid
Left: RMSE (d/o 5 to 2190). Right: Log RMSE (d/o 360 to 2190).

The results show that while high-degree GGMs generally offer good accuracy, their differences diminish at higher degrees, indicating a convergence in representing the Earth's gravity field. However, improvements are needed to lower residual errors, especially in root mean square error (RMSE). Recent machine learning advancements promise enhancements in modeling geoid undulations by capturing complex relationships not addressed by traditional methods. Three ML algorithms, Support Vector Regression (SVR), Random Forest (RF), and Multilayer Perceptron (MLP), were tested at 5-degree intervals up to degree 2190, with 80% of the data for training and

20% for testing. These models significantly improved predictive accuracy, with MLP consistently outperforming SVR and RF, achieving the lowest RMSE and highlighting the potential of deep learning in refining geoid models.

Figure 2 shows the RMSE of the residuals after the MLP was applied. XGM2019e performed well at lower degrees but exhibited the highest RMSE values at higher d/o. EGM2008, GECO, SSG2, and EIGEN_6C4 show similar results. The best improvement was achieved by GECO with an RMSE of 0.77 at d/o 2125.

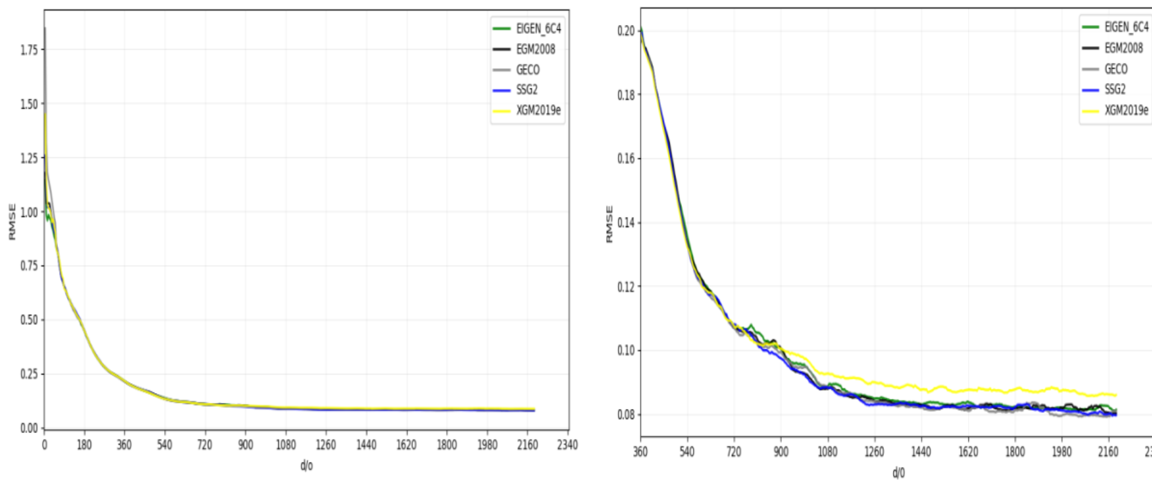


**Fig. 2.** RMSE comparison of Residuals after MLP
Left: RMSE (d/o 5 to 2190). Right: RMSE (d/o 360 to 2190)

Figure 3 shows the RMSE trends before and after applying the MLP. It can be seen that the application of MLP significantly reduced RMSE values across all GGMs for all d/o.

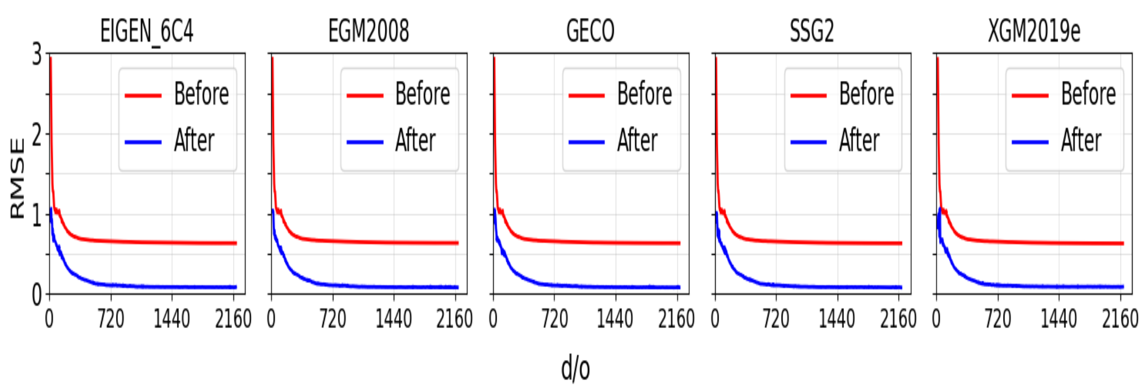


**Fig. 3.** RMSE comparison of residuals before and after applying MLP

Figure 4 also shows the histogram of the residuals before and after the MLP at the d/o that achieved the lowest RMSE after the MLP was applied. It can be seen that MLP reduced the residual spread from approximately ± 1.6 m to less than ± 0.5 m, significantly improving the accuracy of GGM-derived geoid undulations.

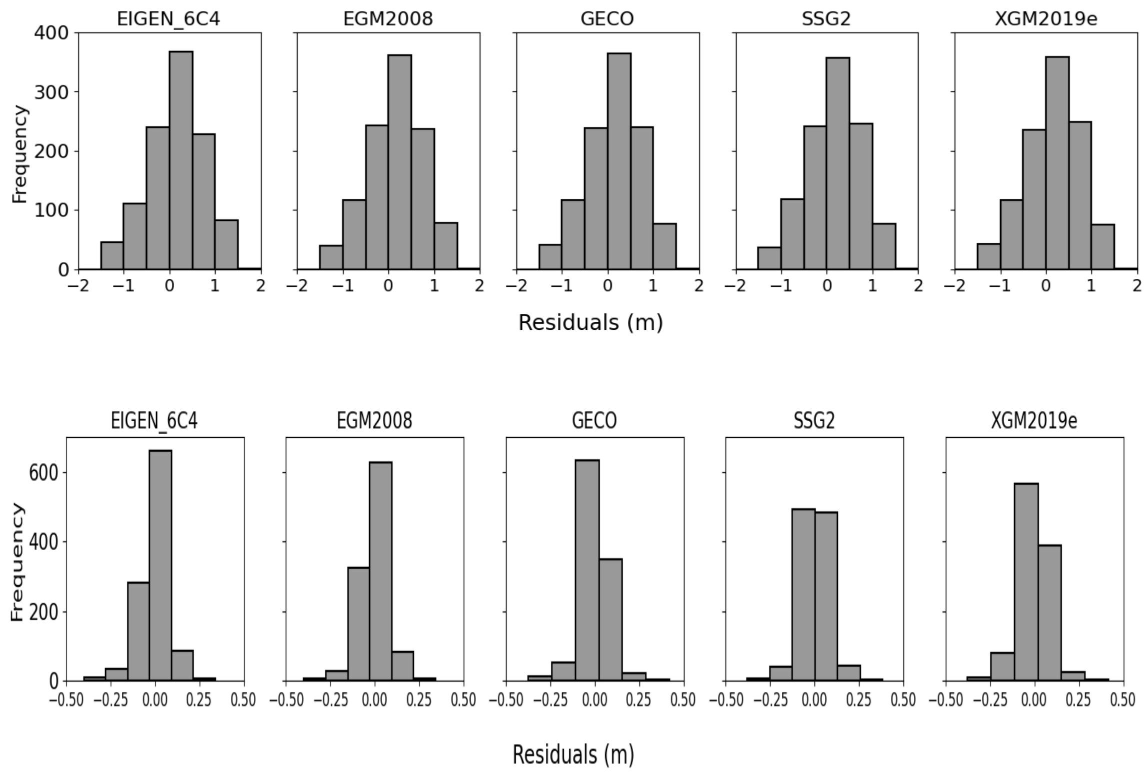


**Fig. 4.** Histogram of residuals before ML (up) and after MLP (down)

Table 1 summarizes the improvement in RMSE for all models achieved with the application of the machine learning techniques.

**Table 1**. Machine learning improvement of GGM geoid undulations

| Model | GGM Best RMSE (cm) | SVR | | RF | | MLP | |
|---|---|---|---|---|---|---|---|
| | | Best RMSE | Improvement (%) | Best RMSE | Improvement (%) | Best RMSE | Improve-ment (%) |
| SGG-UGM-2 | 62.93 | 9.44 | 85.00 | 10.81 | 82.82 | 7.73 | 87.72 |
| XGM2019e_2159 | 62.75 | 9.87 | 84.27 | 10.75 | 82.87 | 8.37 | 86.67 |
| GECO | 63.07 | 9.44 | 85.03 | 10.78 | 82.91 | 7.70 | 87.79 |

| Model | GGM Best RMSE (cm) | SVR | | RF | | MLP | |
|---|---|---|---|---|---|---|---|
| | | Best RMSE | Improvement (%) | Best RMSE | Improvement (%) | Best RMSE | Improve-ment (%) |
| EIGEN-6C4 | 63.09 | 9.61 | 84.77 | 10.80 | 82.88 | 7.96 | 87.38 |
| EGM2008 | 63.31 | 9.37 | 85.20 | 10.66 | 83.17 | 7.81 | 87.66 |

In conclusion, this study shows that while increasing the degree and order of GGMs improves resolution, it can introduce high-frequency noise that may decrease accuracy. For example, at a maximum degree of 2190, RMSE values for SGG-UGM-2 increased by 9.46 cm for SVR, 11.28 cm for RF, and 7.78 cm for MLP, indicating that the optimal truncation degree varies by model. The research highlights the effectiveness of machine learning, especially the Multilayer Perceptron, in enhancing geoid undulation predictions. By combining traditional geodetic methods with data-driven approaches, this study provides a solid framework for improving GGM accuracy. Future work should include more gravity, altimetry, and GNSS data to further enhance GGM-based geoid models.

***References:***


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